RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.Sc. SECOND SEMESTER (January – June) 2015

Mid-Semester Examination, March 2015

Date : 18/03/2015 **MATHEMATICS (Honours)**

Time : 11 am – 1 pm Paper : II Full Marks : 50

[Use a separate answer book for each group]

Group – A

1. Answer **any one** : [1×4]

   a) If $a_1, a_2, ...a_n$ be n positive real numbers then prove that $\frac{a_1+a_2+...+a_n}{n} \geq (a_1a_2...a_n)^{1/n}$.

   b) State and prove De Moivre's theorem for complex number.

2. Answer **any three** : [3×3]

   a) If a, b, c be positive real numbers such that the sum of any two is greater that the third, prove that $a^2(p-q)(p-r)+b^2(q-p)(q-r)+c^2(r-p)(r-q) \geq 0$ for all real p, q, r.

   b) Prove that $(1!)(3!)(5!)...((2n-1)!) > (n!)^n$.

   c) Three complex numbers z_1, z_2, z_3 are such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|$. Prove that they represent the vertices of an equilateral triangle in the complex plane.

   d) If a, b, x are real and $|a+ib|=1$, prove that $(a+ib)^{ix}$ is purely real.

   e) Show that : $\text{Tan}^{-1}(-1+i) = \frac{1}{2}[(2n+1)\pi+\tan^{-1}2]+\frac{i}{4}\log 5$.

3. Answer **any three** : [3×4]

   a) State and prove the Cauchy's condensation test.

   b) Test for convergence of the series : $\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+...$

   c) i) Test the convergence of the following series : $\sin\frac{\pi}{2}+\sin\frac{\pi}{4}+\sin\frac{\pi}{6}+...$

      [Hints : For $0 < x \leq \frac{\pi}{2}, \frac{2x}{\pi} \leq \sin x$] [2]

      ii) Let $\sum a_n$ be a series of positive terms. Let $C_n = n\left(\frac{a_n}{a_{n+1}}-1\right)-1$. If $\{n^q C_n\}_n$ is bounded for some q > 0 then show that $\sum a_n = \infty$ (Using Kummer's test) [2]

   d) Let A be a non-empty subset of $\mathbb{R}$. For each $x \in \mathbb{R}$, we define a number $p_x = \inf\{|x-a| : a \in A\}$. Then the function $f:\mathbb{R}\to\mathbb{R}$ define by f(x) = p_x show that—

      i) f is continuous

      ii) $f^{-1}(\{0\}) = \bar{A}$. [2+2]

Group – B

4. Answer **any two** : [2×5]

   a) Show that a real square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew symmetric matrix.

b) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ show that $A^2 - 4A - 5I_3 = 0$. Hence obtain a matrix B such that $AB = I_3$.

c) Express as the product of two determinants and hence prove that

$$\begin{vmatrix} (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix} = 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a).$$

Answer **any one** : [1×15]

10. a) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax = b$, $x \geq 0$ corresponds to a B.F.S. [6]

b) Solve the following L.P.P :

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to $-3x_1 + 2x_2 + 3x_3 = 8$

$-3x_1 + 4x_2 + 2x_3 = 7$

$x_1, x_2, x_3 \geq 0$ [6]

c) If x_1, x_2 be real, show that $X = \{(x_1, x_2) : 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set. [3]

11. a) Reduce the feasible solution $x_1 = 2$, $x_2 = 1$, $x_3 = 1$ of the system of equations

$$x_1 + 4x_2 - x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 8$$

to B.F.S. [5]

b) Use Charnes M-method, solve the L.P.P

Minimize $Z = 4x_1 + 3x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$x_1, x_2 \geq 0$ [6]

c) Find all the basic solutions of the system

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$ [4]

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